MOST EFFICIENT WAY OF MIXING POWDERS

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1. Introduction. There are two tendencies in the evolution of all systems observed in nature: 1) increase in chaos or disorder and 2) the occurrence of order and self-organization. In the usual case, increase in chaos results in the system forgetting its history and ultimately coming to a state that is independent of the initial data and the history (more exactly, the history of the degradation). That state is stationary and the evolution stops there. One example is familiar to all from his own experience: if special measures are not taken, disorder in one's house can only increase. The special measures to restore order always involve consuming energy and intellectual effort.

Sometimes, one has the opposite task: to provide measures that will accelerate the evolution and transfer the system to the stationary state as rapidly as possible. That intervention also requires effort and energy, although here the purpose is not to induce order but the converse: to attain maximal disorder rapidly. Sometimes, such tasks have practical significance, one of which is mixing powders [1].

We consider this as follows. Let there be two powders (components) A and B. These are dispensed in advance in the necessary recipe ratio. Let A be the key component. By definition, its volume is not larger than of component B. The two components are dry powders consisting of distinct solid particles. The A and B particles may differ in shape, size, and particularly in density. We assume however that if some mixture of A and B is set up, it does not separate spontaneously. The initial task is to locate the materials A and B in a given vessel V in such a way that they are uniformly distributed one in the other. We thus have a discrete mixing process.

There are two ways of handling this. The first is as follows. The necessary number of particles of B is placed in V. Then a particle A is placed between them and so on. One thus obtains a homogeneous packing. As a rule, direct execution of that method is unrealistic because of the enormous number of particles and the small size of them. One can seek technically applicable particle packers. This approach is deterministic.

The second approach essentially differs from the first and is associated with random particle displacements, which lead to mixing. The initial task needs to be defined more closely for a random process. Firstly, one has to define the final purpose: obtaining a homogeneous mixture. Here we have to assume that there exists some minimal volume ω within which the component distribution is not significant. Then the homogeneity requirement means that in any region ω defined in V the ratio of the A and B components should be such as in the entire volume V. More precisely, for any region with volume ω, the probability of deviation from the recipe ratio by more than a permissible ε should be less than some preset value Δ.

The last requirement sets a theoretical limit to the performance of stochastic mixing methods. That limit can be calculated exactly. For example, one can find a lower bound to ω as a function of V, ε, and Δ for given numbers and relative sizes of the A and B particles. On the other hand, ω is determined by technological requirements, and in fact ω is equal to the volume of the sample taken to check the mixing quality. If the numbers of A and B particles in ω are sufficiently large on the basis of the recipe, the probability of impermissible fluctuations becomes negligible. In most cases, this is the situation. We consider only this form in what follows. This means that it is permissible to use stochastic processes to prepare a homogeneous mixture. In the opposite case, for example, when the concentration of the key component is very low, only the deterministic approach is possible.

A large number of particles is an obstacle to implementing the deterministic mixing method. That circumstance makes the stochastic approach realistic. It can be used in a set of different processes, which differ in performance, quality, principles used, and so on.

We then ask whether all the conceivable mixing processes include one that is the best in quality and efficiency. In a word, is there a certain ideal, i.e., something that cannot be exceeded even in principle? We attempt to define this process [2].

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2. Ideal Mixing. We first consider the following fact. It is familiar that if a fragrance is dispersed in a closed room, the odor will be distributed over the entire room after a certain time. If the weight can be neglected, that distribution will be uniform. The reason is that a gas consists of individual molecules, which traverse disordered random paths. Repeated realization of them results in uniform mixing. Therefore, there is in nature a process that leads to ideal mixture quality. We try to use it to solve our problem.

It is clear that the final result is not dependent on the molecule's dimensions. Therefore, if random free paths are realized for powder particles, we obtain a homogeneous mixture, as for gases.

However, there is a substantial difference in behavior between powders and gases. Gas molecules receive momenta from the vibrating atoms in the vessel walls. To produce comparable momenta in the powder particles, the walls of the vessel must vibrate. However, that direct use of the analogy encounters some technical and essential difficulties. Firstly, a large vibration energy is required, and secondly there are the interactions between the particles on collision. The kinetics will be dependent on the particle properties and in general one cannot guarantee mixture quality. In the initial formulation, we were required to define an ideal process that will guarantee quality for any physical properties of the particles. Also, the gravitation force is incomparably more important for powders than it is for gases. For example, in the absence of vibration, the powder particles are in contact and form a certain packing. That packing is close, so there is no space for free particle paths.

Therefore we proceed as follows. We pour the components A and B into V in an arbitrary fashion. The initial distribution will be inhomogeneous. Let t denote time. Let the initial state correspond to $t = t_0$. We identify each particle by a parameter ρ and introduce a cartesian x, y, z coordinate system and use the radius vector r to denote the position of each particle: $\mathbf{r} = \mathbf{r}(\rho, t)$. Then we perform some mixing operations, e.g., we direct the entire mass from V into a separate mixer. We process it there for a time τ and then return it to V. After those steps, all the particles are in V, and they will be at rest and again form a definite packing. Each particle ρ occupies a new position. This is the result of processing the mixture from t_0 to $t_0 + \tau$. Therefore, if we are not interested in the mixture transformations in the interval $t_0, t_0 + \tau$, we can say that in time τ all the particles have remained within V and have been displaced by the vector

$$\mathbf{l}(\rho, t_0) = \mathbf{r}(\rho, t_0 + \tau) - \mathbf{r}(\rho, t_0), \tag{2.1}$$

which may be called the range of particle ρ. It can play entirely the role of the mean free path of a gas molecule.

Then we process the mixture again from time $t_0 + \tau$ to $t_0 + 2\tau$. We again return the material to V and in a state of rest we identify the position of particle ρ in the new packing: $r = r(\rho, t_0 + 2\tau)$. This corresponds to a new particle range and so on. We can thus speak of a sequence of transformations of region V into itself. Here it is convenient to introduce the dimensionless time $N = (t - t_0)/\tau$ and call the integer values of N, the cycle numbers. In N cycles, each particle acquires the resultant displacement

$$\mathbf{L}(\rho, N) = \sum_{i=0}^{N-1} \mathbf{l}(\rho, i) = \mathbf{r}(\rho, N) - \mathbf{r}(\rho, 0). \tag{2.2}$$

Quality will be provided if the mixture tends to a homogeneous state for $N \to \infty$. However, ρ is a random quantity, which is insufficient by itself for this to occur. For example, if the heavy particles have a greater probability of being displaced downwards, an inhomogeneous state will be attained in the limit. We consider this in more detail.

We first introduce the characteristics of the stochastic process. Let F be the probability density for the random quantity l. By definition, the following is the probability that vector l has a length between S and s + ds and lies within a cone having solid angle $2d\Omega$ and having its axis directed along a given unit vector e, which is equal to the product of F by the corresponding volume element: $\pi s^2 F ds (d\Omega)^2$

Function F alone is insufficient to characterize the process because the particle ranges are not simply random quantities but form a random vector field. For example, if we know that a certain particle is displaced to a certain point in space, the conditional probability of displacement of any other particle to that point should be zero. On the other hand, there is unit probability that there are particles displaced to the immediate neighborhood of that point. In other words, after any realization of the random l field, there should be a reasonable number of contacts of each particle with its neighbors. For particles at the free surface, the packing should be such that the maximum stacking angle α does not exceed the limiting value φ (φ is the natural piling angle of the material). Consequently, V is transformed into itself apart from the outline of the free surface, which may change from cycle to cycle. This feature and the field properties of the displacements complicate all the theoretical constructions.
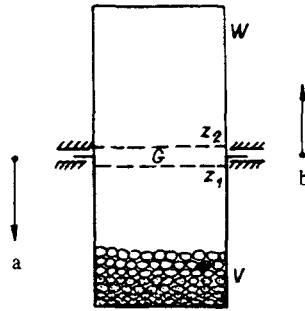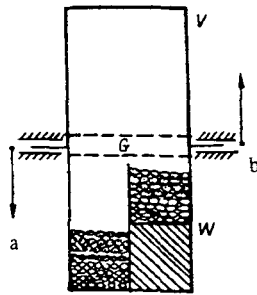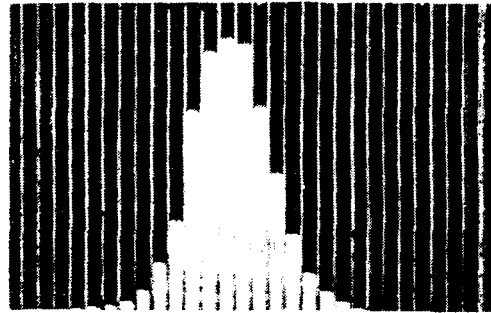
Fig. 1

Fig. 2

Fig. 3

To describe 1 as a random field, one needs to introduce multidimensional probability densities [3]. The above specifications for the random field will be called the compatibility condition. That condition for F and the multidimensional probability densities mean that any particular realization of the random field of ranges should be such that a particle packing is transformed into a packing. It is complicated to give an analytic formulation of that condition (this difficulty is overcome in another way below). Here it is sufficient to note that it occurs.

The collective particle behavior may be expressed not only in obedience to the compatibility condition; closer connections can exist. The main one is that nearby particles have a tendency to remain nearby after random-displacement realization. For example, let a mixing operation amount to single shaking of vessel V. It is clear that here the particles will occupy new positions, but closely spaced particles as a rule remain closely spaced. The compatibility condition will be met.

Now the initial task can be formulated more definitely: we can reduce its solution to three questions. 1. What conditions must F and the multidimensional probabilities satisfy in order that the mixture tends to a homogeneous state as time passes? 2. Under what conditions will the transition to the homogeneous state be completed in a shorter time? 3. How can one realize a process that will correspond to probability densities with the required features? There are various mathematical difficulties in a rigorous examination of these questions, so we first consider some intuitive arguments.

We first establish what arguments may govern F. Firstly, in general F may be dependent on the particle type (density and other physical properties) and then on the particle coordinates $r(\rho, N)$. There is a natural dependence on the random displacement s and the correction of it e. In principle, there may also be an explicit dependence on the time N (this will be the case if the mixture processing method alters from cycle to cycle). F may also be dependent on some other factors. Then in general

$$F = F(\rho, r, s, e, N,...).$$

It appears obvious that if F is dependent on the particle type, then in the limit the mixture will be not homogeneous but separated in accordance with the feature that influences the probability density. Therefore, as first condition we assume that all the probability densities should not be dependent on the particle type in an ideal mixing process.

We now consider the resultant displacement of (2.2). Let N^* be the instant when the mixture attains the homogeneous state. Then $L(\rho, N^*)$ is the over-all displacement of a particle from the initial position to the final one. We can use the initial
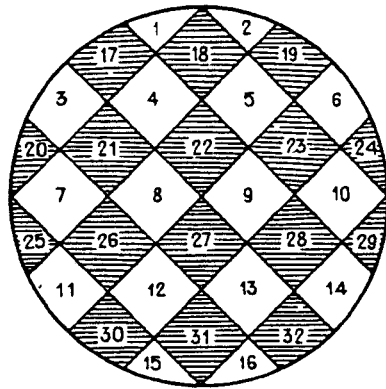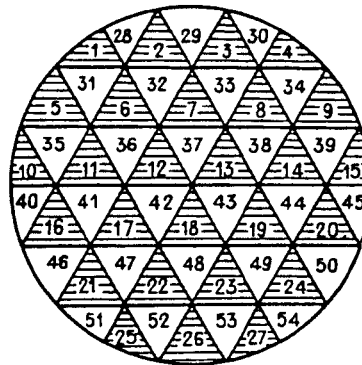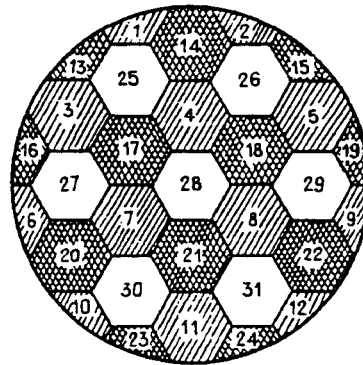
Fig. 4

Fig. 5

Fig. 6

radius vector r^0 to identify the particle ρ. It is readily understood that the function L of r^0 does not have any a priori smoothness conditions. Moreover, in general it will be everywhere discontinuous. For example, let all the particles of the key component be collected together at the start in some part of V, i.e., at the initial instant, each A particle (apart from boundary ones) is in contact only with type A particles. In the final state, on the other hand, if the concentration of A is low, each A particle should be surrounded only by type B ones. This means that originally close particles should correspond to widely differing L. Or in other words, closely spaced particles sooner or later should be separated in the mixing. However, if there are close correlations in the behavior of the different ensemble particles, considerable time will be required for that separation. On the other hand, there should be a certain collective behavior associated with the compatibility at all times. We thus conclude that the field of random displacements in an ideal process should satisfy only the compatibility conditions, and in all other respects the behavior of the different particles should be independent.

We return to the analogy with gases. If V contains a gas with normal density, the mean free path is extremely limited. For example, paths comparable with the size of the entire vessel are practically ruled out. This circumstance is the main constraint on the mixing performance. The situation is opposite for a powder. Here the particles fill the volume completely and there is no free space for paths. However, the entire process and the path concept of (2.1) are defined in such a way that I can take virtually any value. For example, if a particle was at one wall of the vessel, after one mixing cycle it can quite well be at the other wall.

Further, homogeneity in the mixture means that the properties are the same at all points and in all directions. In other words, the properties are invariant with respect to transport and rotation of the coordinate system. One assumes that the functions that control the effective transition to that state should be invariant. For F, this means that it is independent of the arguments s and e. However, the mixing occurs in a restricted volume, and all the particles must meet the condition for not penetrating into the walls. This means that F should in fact be dependent on e and s. Therefore, the invariance requirement must be refined: in an ideal mixing process, the probability density for a random displacement l should be dependent on the

position of the particle **r**, the value of *s*, and the direction of the random displacement **e** only to such an extent as is required by the condition for nonpenetration of the wall, and also as the constraint on the free-surface configuration. That condition has a simple analytic formulation:

$$F(\rho, \mathbf{r}, s, \mathbf{e}, ...) \equiv \text{const} \quad \text{for} \quad \mathbf{r} + s\mathbf{e} \in V,$$
$$F(\rho, \mathbf{r}, s, \mathbf{e}, ...) \equiv 0 \qquad \text{for} \quad \mathbf{r} + s\mathbf{e} \notin V. \tag{2.3}$$

Condition (2.3) is equivalent to the following procedure for a continuous medium. Let V be filled with an arbitrary mixture of two media having density γ. We split it up into V/ω parts and stretch each of them uniformly to V. The density will be $\gamma\omega/V$. Then we embed all these parts into one another. The density returns to the previous value γ. It is clear that for $\omega \to 0$, that procedure will give a homogeneous distribution for any initial state. Then (2.3) constitutes the probability analog of that procedure for a medium consisting of discrete particles.

Then for a mixing process to be maximally effective as regards quality and rate, it is necessary and sufficient to meet the following three conditions: a) all the probability densities for the random displacement of the particles should not be dependent on the particle properties; b) the multidimensional probability densities should be such that only the displacement compatibility condition is met, and in other respects the behavior of the different particles, including neighbors, should be independent; and c) the one-dimensional probability density should be dependent on the particle position, and on the magnitude and direction of the random path only to the extent required by the condition that the particles do not penetrate the wall and by the constraint on the free surface configuration.

Conditions a-c give a description of the ideal process in the formal language of probability densities. We then consider deriving the constructive features of the ideal process.

3. Principles For Realizing Ideal Mixing. We have previously considered the positions of particles at discrete instants $t = 0, 1, ...$ i.e., we have in fact examined only the results from processing after each mixing cycle. A constructive description requires us to examine the behavior of the material within each cycle. We denote by **l** the displacement of a particle identified by parameter ρ:

$$\mathbf{l} = \mathbf{r}(\rho, 0) - \mathbf{r}(\rho, t). \tag{3.1}$$

For $0 < t < 1$, this is the displacement within the first cycle. We define the displacements for the second cycle similarly and so on. For $t \neq 0, 1, ...$, the material may be outside vessel V, so in general $\mathbf{r}(\rho, t) \notin V$. Then the task can be formulated as follows: what should be the (3.1) displacements to ensure that the result from each cycle corresponds to conditions a-c?

Here one obvious argument applies. We assume that the (3.1) displacements within a cycle satisfy certain constraints. We will use only the above sufficient condition. In other words, we impose constraints a-c not only on the (2.1) displacements but also on the (3.1) ones. This enables one to distinguish certain constructive features of the process.

According to condition *a*, the particle displacements should be independent of the type. In nature, there is a unique nontrivial process in which the kinematics of the motion will be independent of the weight, size, and other particle characteristics: free fall in a gravitational field. This means that the particles must be able to fall freely (this is a central point in all the constructions). Condition *a* also implies that the mixing should be performed in vacuum for light particles or high rates.

We now consider the collective particle behavior. Condition b requires that the displacements of any particle should be independent within the compatibility limits. Connectedness in the behavior of widely separated particles is an exotic feature. In practice, connectedness is based only on direct contacts between particles. Therefore, we are concerned with a process in which one would have independent displacement of adjacent particles. However, if there are contacts between them, the behavior will necessarily be dependent. Therefore, in an ideal process, the mixture must be distributed in space in such a way that the contacts vanish and independent relative displacements between particles become possible.

The theoretical description of the ideal case thus predetermines the following basic features of the real process: 1) the particles must be able to fall freely; 2) the mixture must be distributed in space in such a way that the contacts between adjacent particles vanish; 3) random velocity components must be applied during the particle fall; and 4) any upward transport, if there is a need for it, must amount only to displacement of the specimen as a rigid whole (this is a trivial case when the kinematics will be independent of the particle properties).

The next natural question is how such a process can be realized? In principle, there may be a set of technical solutions that approximate to some extent to the ideal. We consider one of them.

4. Technical Realization Example. It is familiar that any pouring operation with a powder mixture leads to separation as a consequence of segregation. Therefore, a pouring operation is best excluded. This means that the vessel V (store) in which the homogeneous mixture must lie should be part of the mixer. We have seen above that the mixing proper should be performed under conditions of free fall. This imposes a constraint on the shape of the store: the side surface must be obtained by the motion of a vertical generator. We assume for definiteness that V is a right circular cylinder. To distribute the mixture in space, we supplement the store to form a cylindrical container (Fig. 1). We assume that the vertical size of the container is bounded, with the result that the time for a single particle to fall over the height is insufficient for complete mixing. Then one needs to organize a periodic process, which includes transporting the material upwards. As condition a can be met only with the mixture moving downwards, there must be no relative particle displacements during the upward transportation. That transportation can be provided by rotating the container suddenly around a horizontal axis. The speed of rotation should be such that the centrifugal force prevents the mixture from premature pouring. That speed is readily calculated. It is more difficult to give a rigorous calculation for the braking state at the top point. Inertial forces occur during braking, which may lead to loss of stability in the material. Therefore, the braking should not be very sharp (this constraint can be eliminated if the material is covered above by a slide).

Then after container rotation, the mixture is at the top point and then falls freely. The arrows in Fig. 1 denote the direction of the gravitational force (a initial state and b state after rotation). It is clear that free fall still does not lead to mixing. If the specimen is displaced in a self-similar fashion from position b, it will be displaced downwards as a rigid whole. Of course, in practice there may be relative particle displacements on account of interactions with the walls and there may be other relevant factors. However, to assume them would be essentially incorrect. Such displacements must be considered only as perturbations in an ideal process. Therefore, the displacements must be specified in some forced fashion in order to provide a definite guarantee of randomness.

We consider a state in which the horizontal component is determined by one device and the vertical one by another. Only the horizontal component is applied during the fall. The corresponding device is denoted symbolically by G (Fig. 1). Let · it occupy the layer $z_1 \leq z \leq z_2$. If a particle enters device G at a point having the horizontal coordinates (x, y), it leaves G with other coordinates ξ and η (ξ and η are random quantities). In space, this corresponds to the particle passing from point (x, y, z_1) to point (ξ, η, z_2), and the order in which the particles follow along the vertical is unaltered: if two particles enter G in sequence with some definite vertical separation, they leave G with new horizontal coordinates but the previous vertical separation.

The result from the operation of such a mixer is obvious. On the first rotation of the container, the particles pass through G and enter the part of the container W. Then on the second rotation, they pass again through G and are again in vessel V. The result of this is that each particle obtains only a horizontal displacement. Therefore, any vertical inhomogeneity in the mixture will not be eliminated by that mixer. The reason for this is clear and lies in the design of device G. However, it can be modified readily, and one can propose a method of averaging the mixture along the vertical.

In fact, the reason for the lack of a vertical component can be interpreted as follows: particles that arrive in part W earlier will return to G later after rotation. The order is rigorous and is independent of the displacement randomness in the horizontal plane. Therefore, that order must be altered artificially in order to obtain a vertically random component. In that formulation, the following solution is natural. We divide up vessel W with vertical walls to give cells differing in depth (Fig. 2). Then particles that are the first to arrive in the set of cells will on subsequent rotation return to device G not necessarily last. Everything will be dependent on the depth of the cells the particles enter. However, the cell depth in turn is dependent on the position in the horizontal plane, which is a random position for a particle, so this leads to a vertical random displacement component.

These cells differing in depth also resolve the problem of dispersing the mixture in space. When the container is rotated to position b (Fig. 2), the first material to arrive at device G is from the shallowest cell, and then that from the next deepest one, and so on.

The essential scheme for the technical solution is thus clear. We now consider it in more detail. It has been shown above that the particles, no matter what their type, should be given set velocities, not momenta or energies. Therefore, the working parts of device G should be such that the kinematics of the motion will be independent of the number and properties of the particles with which they interact. There are various forms that can be taken for the drive within the container, e.g., vertical plates vibrating or rotating in various directions, while the drive may be provided by the body of the mixer itself. In fact, a particle when falling is not linked to the body, so relative displacement is obtained if accelerated motion is applied not to the particle but to the body (for example, torsional oscillations in the middle of the mixer). Such oscillations are quite suffi-

cient to move the working organs in device G. They may be designed as elastic elements fixed at both ends. Then they will oscillate as does the body. A larger amplitude and higher frequency may be obtained if the elements are mounted only at one end. The oscillation parameters will be dependent on the elastic characteristics of the elements, one-side amplitude constraints, and so on.

In one form, device G may be built as a set of randomly packed wires or metal strips. In that case, the body oscillations provide the stimulus for the powder to pass through the labyrinth. Vibrational action on the particles is permitted only during the downward motion. When the particles rest on one another, the action must cease, since the vibrational packing adversely affects the homogeneity. Therefore, the parts V and W in which the material accumulates should not be subject to vibration.

The horizontal-mixing unit is thus one with numerous parameters. How are they to be optimized? Direct experimental study is very laborious because of the difficulty in analyzing the mixture quality and the dependence of the final result on various factors. Therefore, we need a separate performance criterion for unit G, which is used in optimizing it.

This can be done as follows. We denote by P the probability density of the coordinates ξ and η, namely the probability is as follows — that a particle that has entered device G at a point with horizontal coordinates x and y will leave it in the $d\xi$ $\times$ $d\eta$ region of points ξ and η: $P(x, y, \xi, \eta)d\xi d\eta$). This function P can be determined by experiment. To do this, we place under G a planar set of cells separated by vertical walls (indicator, Fig. 3). We supply a thin jet of the material (δ function) to point (x, y). Then particles displaced by G to the neighborhood of point (ξ, η) enter the corresponding cell in the indicator and accumulate there. The level distribution curve over the cells can be observed visually (glass front wall for the indicator), and that curve will be a section of the $P(x, y, \xi, \eta)$ surface for the given x and y. Here a simple fact must be noted. The experiments give directly a graph in axes having linear scales. The slope of the graph is proportional to the time for which the jet is supplied. Therefore, the supply time can always be chosen such that P is determined quite accurately. For the experimental curve to be taken as a section of surface P, the scale along the vertical axis must be normalized. When we have information on P, we can trace how the performance of the device varies when one alters the different parameters, constructive components, and so on. The dependence on x and y enables one to estimate the effects from the side walls. One can compare the P for various materials to determine how sensitive the device is to the particle type.

The particle interaction within G (linearity) is important to the theory. We perform the following experiments. The jet of material is supplied in turn to the points (x_1, y_1), (x_2, y_2), which gives the corresponding graphs P_1 and P_2. We then supply the material simultaneously to those points. The result is P_{12}. If $P_{12} = P_1 + P_2$ (for the curves without normalization), interaction can be neglected (linearity). If the equation does not apply, there is interaction and the process is nonlinear. The type of nonlinearity can be established from similar experiments for three or more points.

What criterion is used in evaluating the operation of G? Conditions a and b imply that an ideal device works in such a way that P is independent of the particle type and the coordinates, i.e., $P \equiv const$. This means that the indicator cell should be filled uniformly in all the experiments. The performance of a real device is therefore evaluated from the deviation of P from a constant.

One can consider the theoretical determination for all the probability densities from the known construction of a device and its parameters. For example, if the device is a bundle of randomly installed wires, the densities can be determined by methods from the theory of random walks [4].

We now consider the operation of the horizontal-mixing device from a somewhat different viewpoint. We assume that the input has a particle distribution characterized by $\varphi_0(x, y)$, and the output is the new distribution $\varphi_1(\xi, \eta)$. The operation can be considered as that of the operator G on the function φ_0: $G\varphi_0 = \varphi_1$. Specifying operator G is equivalent to complete description of all the probability densities P_1, P_{12}, ... The law of conservation of matter implies

$$\int_S G\varphi dS = \int_S \varphi dS \qquad (4.1)$$

(S is the horizontal cross section). The ideal device corresponds to an operator that converts any function to a constant: $G\varphi \equiv C$. The $G\varphi \not\equiv C$ for a nonideal device. However, if we make the transformation sequence $G...G = G^n$, the distribution should tend to be uniform, $G^n\varphi \rightarrow C$, as n increases, which can be used to estimate the performance of G. For example, we can take the performance norm as $0 < 1/n \leq 1$, where n is the least of the numbers for which $G^n\varphi$ can be taken as constant. From $\lim_{n \to \infty} G^n\varphi = C$ it follows that a uniform flow of particles after passage through G should be converted to a uniform one (homogeneity condition):

$$GC = C. \qquad (4.2)$$

The mixture settings are of some interest, when operator G is known for a certain height of the device ($z_2 - z_1$) (e.g., the height of the packet of randomly spaced wires), and it is required to determine the height that provides $G\varphi \approx \text{const}$, which is not necessarily a multiple of $z_2 - z_1$.

We note that device G can always be made to operate efficiently. If it works inefficiently, it is necessary to place n devices in sequence along the vertical. The performance of the random horizontal-displacement device is thus a question related to the height. We subsequently assume that all the characteristics of device G are known in advance from experiments with indicator cells.

We now consider some experimental results.[*] Figure 3 shows the particle distribution obtained with a jet of dry quartz sand. G was a packet of randomly arranged copper wires 0.5 mm in diameter, height 30 mm, and relative density 6%. The distribution was almost normal. An interesting point was that the slope of the normalized profile was dependent on the particle velocity: higher velocities resulted in a somewhat flatter curve. The experiments showed that particle interaction was not important even when the flow was not very widely spaced, which means that the corresponding operator G is virtually linear.

We now consider block W in more detail, which provides vertical mixing. All the parameters are geometrical, and include the shape and transverse dimensions of the cells, the depths, and the order of their placing. Unit W works quite deterministically. The randomness in the vertical particle range component is entirely governed by the fact that the horizontal component is random. One therefore expects that the optimum parameters of W are related to the probability characteristics of the horizontal mixing device. For example, it is almost obvious that if device G works ideally ($G\varphi = \text{const} \ \forall \ \varphi$), then the order of the cells in space has no significance. Such features makes it difficult to perform a rigorous study on optimizing W, so we consider only approximate criteria.

One of the purposes of the vertical mixing device is to separate the initially closely spaced particles. There is always the chance that particles that were close together on emergence from the horizontal-mixing device will fall into adjacent cells with differing depths, which means that their subsequent paths completely diverge. Therefore, the longer the boundary between cells (and the smaller the dimensions), the more effective the device. On the other hand, the transverse dimensions of the cells are restricted by the condition for free movement of the material from them. These two circumstances enable one to determine readily what the transverse dimensions should be.

Further, the cell shape is a problem in dividing a given region (circle) into subregions. There are technological difficulties in realizing exotic subdivisions, so we consider particular cases where certain simple subdivisions are specified in advance (square, triangular, and hexagonal ones). By k we denote the total number of cells and enumerate them in order of increasing depth: $1 = h_1 \leq h_2 \leq h_3 \leq \ldots \leq h_k = h_{max}$. Here the depth of the shallowest cell is taken as unit (its absolute value should be such that it does not overflow).

The cell depth spectrum can be chosen from the condition for the most uniform dispersal of the mixture in space. If certain cells are identical in depth, the material from them arrives at device G simultaneously. Consequently, greater spread will be provided by a uniform spectrum: $h_{i+1} - h_i = \text{const}$, and we consider only that form.

Then the basic topic is how to locate the cells, i.e., where to place the deepest cell, what depth should the cell have that is placed adjacent to it, and so on? For closely spaced particles, there is much more scope for them to enter different cells in contact on their faces than for cells in contact on edges. Therefore, we naturally assume that the cell array should be such that the average height of a step between cells is the largest:

$$R = \frac{1}{M} \sum_{i=1}^{M} |h_i^+ - h_i^-| \rightarrow \max. \tag{4.3}$$

Here M is the total number of internal faces, while h_i^+, h_i^- are the depths of the cells having a common face with number i. It is found to be impossible to produce substantial differences in the depths of all the cells surrounding a certain given one. Therefore, we assume in the absence of a general proof that a large value for R can be provided by reducing the differences in depth between cells in contact along edges. We reduce the remaining arbitrary element as follows. We establish what is the least number of colors required to color the division in such a way that adjacent cells have different colors. Two colors clearly suffice for the divisions shown in Figs. 4 and 5. We divide the cell depth spectrum into two parts. The white cells relate to

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the first part of the spectrum (group of shallow cells), and the black ones to the second part (group of deep cells). The above conditions are sufficient for one to choose particular arrays.

The array of hexagonal cells (Fig. 6) is determined similarly. Three colors suffice to color a hexagonal array. We divide the cell depth spectrum into three groups. Cells of a given color should always have numbers from a single group. Figures 4-6 show examples that satisfy these requirements (the numbers are those of the corresponding cells).

The next aspect is the absolute difference in cell depth h_{max}. We assume that the horizontal mixing device is homogeneous, i.e., (4.2) applies. Then when the material has arrived from all cells, the free surface will be horizontal. However, it will not be so at intermediate times. For example, if one of the cells is placed much closer than the others, the material from it initially arrives first at the horizontal mixing device and may give rise to a profile of the type shown in Fig. 3. In that profile, the current value of the angle of steepest piling $\alpha(t)$ may exceed the angle of natural pile for the material φ, which leads to the material flowing and thus to segregation. We apply the following constraint to the mixer parameters to eliminate segregation:

$$\alpha(t) < \varphi. \tag{4.4}$$

If the parameters of the horizontal mixing device have been defined along with the cell array, (4.4) determines the upper bound to h_{max} unambiguously.

Then on the whole, the design principles for an ideal mixer are clear. However, this is still insufficient for the realization of an actual device. A real mixer will always have small but finite deviations from any preset conditions of equality type. For conditions of inequality type, the situation is simpler, since they can be met always exactly. We have to consider the consequences of these small deviations and whether the basic parameters of the mixer will be stable under perturbations (the mixer goes over to a homogeneous state and what is the transition time). A numerical study of the kinetics shows that the above principles in fact provide complete mixing, and, the most important point, that the process is stable under perturbations [5]. This shows that the results can be used to produce actual efficient powder mixers [6].

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